OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education**
**Advanced General Certificate of Education**

# MATHEMATICS 4729

Mechanics 2

Wednesday **22 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

**TIME** 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

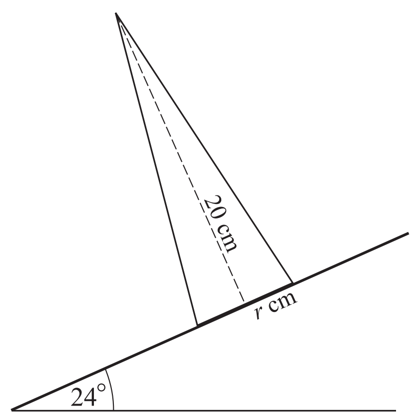
- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g$ m s$^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

**This question paper consists of 4 printed pages.**

**1**




A uniform solid cone has vertical height 20 cm and base radius $r$ cm. It is placed with its axis vertical on a rough horizontal plane. The plane is slowly tilted until the cone topples when the angle of inclination is 24° (see diagram).

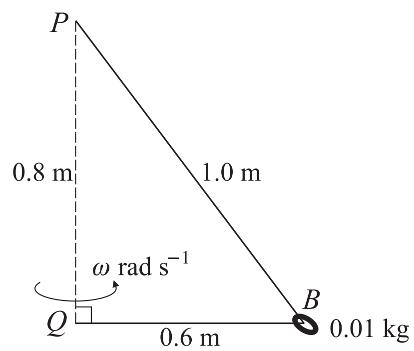
**(i)** Find $r$, correct to 1 decimal place. [4]

A uniform solid cone of vertical height 20 cm and base radius 2.5 cm is placed on the plane which is inclined at an angle of 24°.

**(ii)** State, with justification, whether this cone will topple. [1]

**2** A particle is projected horizontally with a speed of $6 \text{ m s}^{-1}$ from a point 10 m above horizontal ground. The particle moves freely under gravity. Calculate the speed and direction of motion of the particle at the instant it hits the ground. [6]

**3**



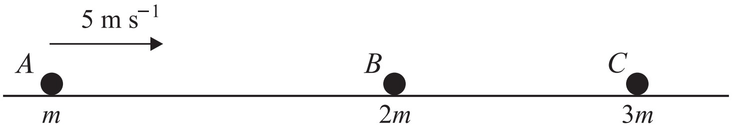

One end of a light inextensible string of length 1.6 m is attached to a point $P$. The other end is attached to the point $Q$, vertically below $P$, where $PQ = 0.8$ m. A small smooth bead $B$, of mass 0.01 kg, is threaded on the string and moves in a horizontal circle, with centre $Q$ and radius 0.6 m. $QB$ rotates with constant angular speed $\omega \text{ rad s}^{-1}$ (see diagram).

**(i)** Show that the tension in the string is 0.1225 N. [3]

**(ii)** Find $\omega$. [3]

**(iii)** Calculate the kinetic energy of the bead. [2]

**4**




Three smooth spheres $A$, $B$ and $C$, of equal radius and of masses $m$ kg, $2m$ kg and $3m$ kg respectively, lie in a straight line and are free to move on a smooth horizontal table. Sphere $A$ is moving with speed $5\text{ m s}^{-1}$ when it collides directly with sphere $B$ which is stationary. As a result of the collision $B$ starts to move with speed $2\text{ m s}^{-1}$.

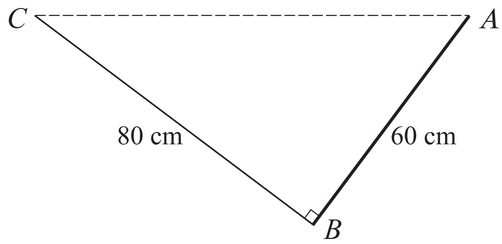
**(i)** Find the coefficient of restitution between $A$ and $B$. [4]

**(ii)** Find, in terms of $m$, the magnitude of the impulse that $A$ exerts on $B$, and state the direction of this impulse. [2]

Sphere $B$ subsequently collides with sphere $C$ which is stationary. As a result of this impact $B$ and $C$ coalesce.

**(iii)** Show that there will be another collision. [3]

**5**




A uniform rod $AB$ of length 60 cm and weight 15 N is freely suspended from its end $A$. The end $B$ of the rod is attached to a light inextensible string of length 80 cm whose other end is fixed to a point $C$ which is at the same horizontal level as $A$. The rod is in equilibrium with the string at right angles to the rod (see diagram).

**(i)** Show that the tension in the string is 4.5 N. [4]

**(ii)** Find the magnitude and direction of the force acting on the rod at $A$. [6]

**6** A car of mass 700 kg is travelling up a hill which is inclined at a constant angle of 5° to the horizontal. At a certain point $P$ on the hill the car's speed is $20\text{ m s}^{-1}$. The point $Q$ is 400 m further up the hill from $P$, and at $Q$ the car's speed is $15\text{ m s}^{-1}$.

**(i)** Calculate the work done by the car's engine as the car moves from $P$ to $Q$, assuming that any resistances to the car's motion may be neglected. [4]

Assume instead that the resistance to the car's motion between $P$ and $Q$ is a constant force of magnitude 200 N.

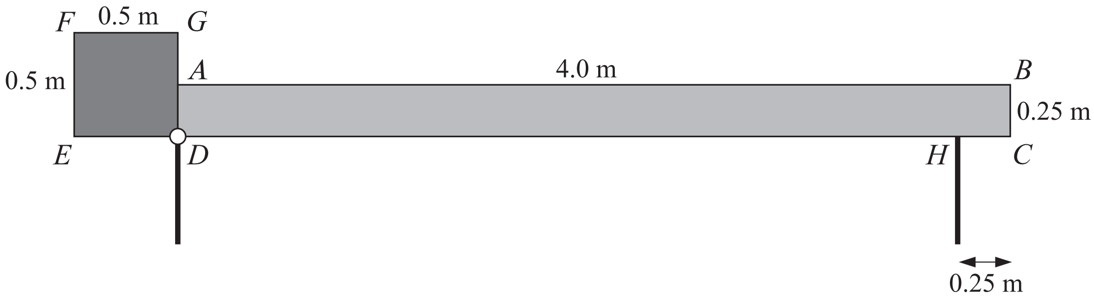
**(ii)** Given that the acceleration of the car at $Q$ is zero, show that the power of the engine as the car passes through $Q$ is 12.0 kW, correct to 3 significant figures. [3]

**(iii)** Given that the power of the car's engine at $P$ is the same as at $Q$, calculate the car's retardation at $P$. [3]



**[Turn over**

**7**




A barrier is modelled as a uniform rectangular plank of wood, *ABCD*, rigidly joined to a uniform square metal plate, *DEFG*. The plank of wood has mass 50 kg and dimensions 4.0 m by 0.25 m. The metal plate has mass 80 kg and side 0.5 m. The plank and plate are joined in such a way that *CDE* is a straight line (see diagram). The barrier is smoothly pivoted at the point *D*. In the closed position, the barrier rests on a thin post at *H*. The distance *CH* is 0.25 m.

**(i)** Calculate the contact force at *H* when the barrier is in the closed position. [3]

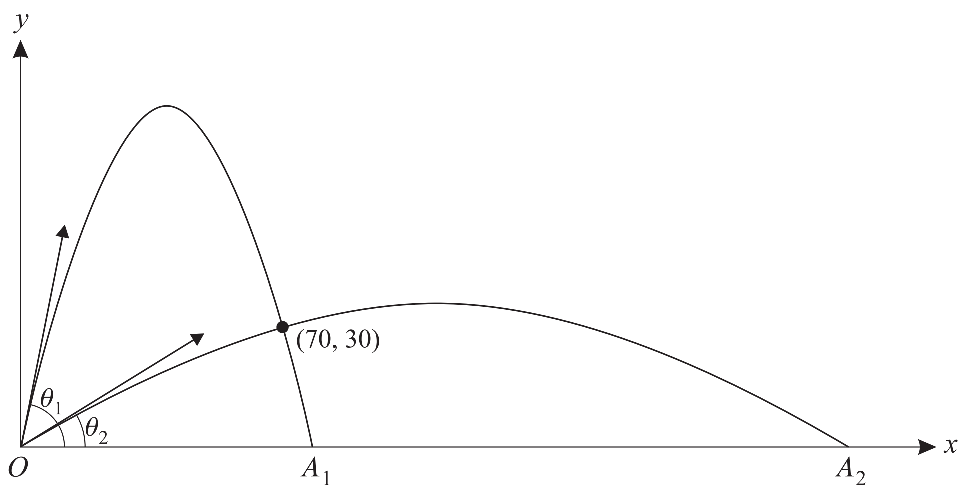
In the open position, the centre of mass of the barrier is vertically above *D*.

**(ii)** Calculate the angle between *AB* and the horizontal when the barrier is in the open position. [8]

**8** A particle is projected with speed 49 m s$^{-1}$ at an angle of elevation $\theta$ from a point *O* on a horizontal plane, and moves freely under gravity. The horizontal and upward vertical displacements of the particle from *O* at time *t* seconds after projection are *x* m and *y* m respectively.

**(i)** Express *x* and *y* in terms of $\theta$ and *t*, and hence show that

$$y = x\tan\theta - \frac{x^2(1+\tan^2\theta)}{490}.$$ [4]




The particle passes through the point where $x = 70$ and $y = 30$. The two possible values of $\theta$ are $\theta_1$ and $\theta_2$, and the corresponding points where the particle returns to the plane are $A_1$ and $A_2$ respectively (see diagram).

**(ii)** Find $\theta_1$ and $\theta_2$. [4]

**(iii)** Calculate the distance between $A_1$ and $A_2$. [5]

| | | | | | | |
|---|---|---|---|---|---|---|
| **1** | (i) | use of h/4 | B1 | | | |
| | | com vert above lowest pt of contact | B1 | | can be implied | |
| | | r = 5 x tan24° | M1 | | | |
| | | r = 2.2 | A1 | 4 | 2.226 | |
| | (ii) | No & valid reason (eg 24°☝26.6°) | B1√ | 1 | √Yes if their r ☜ 2.5 | **5** |

| | | | | | | |
|---|---|---|---|---|---|---|
| **2** | | $v^2$ = 2x9.8x10 | M1 | | energy:½m$v^2$=½m$u^2$ + mgh | |
| | | v = 14 | A1 | | ½$v^2$ = ½.36 + 9.8x10 | |
| | | speed = √($14^2 + 6^2$) | M1 | | (must be $6^2$) $v^2$ = 36+196=232 | |
| | | speed = 15.2 ms$^{-1}$ | A1 | | | |
| | | tanθ = 14/6 | M1 | | cos$^{-1}$(6/15.2) etc | |
| | | θ=66.8°(below) horiz. | A1 | 6 | or 23.2° to the vertical | **6** |

| | | | | | | |
|---|---|---|---|---|---|---|
| **3** | (i) | Tcosθ = 0.01 x 9.8 | M1 | | resolving vertically | |
| | | 8/10T = 0.01 x 9.8 | A1 | | with cosθ = 8/10 | |
| | | T = 0.1225 N | A1 | 3 | AG | |
| | (ii) | T + Tsinθ = ma | M1 | | resolving horizontally | |
| | | use of mr$\omega^2$ | M1 | | | |
| | | ω = 5.72 rads$^{-1}$ | A1 | 3 | | |
| | (iii) | K.E.=½x0.01x$(r\omega)^2$ | M1 | | ½m$v^2$ with v=rw | |
| | | K.E.= 0.0588 | A1√ | 2 | √0.0018 x their $\omega^2$ | **8** |

| | | | | | | |
|---|---|---|---|---|---|---|
| **4** | (i) | 5m = mu + 4m | M1 | | cons. of mom. | |
| | | u = 1 | A1 | | | |
| | | e = (2-1)/5 | M1 | | | |
| | | e = 🗐 | A1 | 4 | | |
| | (ii) | I = 4m | B1 | | | |
| | | → | B1 | 2 | to the right | |
| | (iii) | 4m = 5mv | M1 | | | |
| | | v = 😐 | A1 | | | |
| | | 😐 < 1 | B1 | 3 | | **9** |

| | | | | | | |
|---|---|---|---|---|---|---|
| **5** | (i) | 60T = 15x30cosθ | M1 | | moments about A | |
| | | " | A1 | | | |
| | | 60T = 15x30 x 0.6 | A1 | | cosθ = 0.6 | |
| | | T = 4.5 N | A1 | 4 | **AG** | |
| | (ii) | X = Tsinθ | M1 | | res. horiz. (or moments) | |
| | | X = 3.6 N | A1 | | | |
| | | Y + Tcosθ = 15 | M1 | | res. vert.(3 terms) (or moments) | |
| | | Y = 12.3 N | A1 | | | |
| | | R = 12.8 N | A1√ | | √(their $X^2 + Y^2$) | |
| | | 73.7° to horizontal | A1√ | 6 | or 16.3° to vert.√tan$^{-1}$their(Y/X) | **10** |
| | | **or triangle of forces**: Triangle (M1) $R^2$ =$15^2$+$4.5^2$-2x4.5x15x0.6(M1A1) | | | | |
| | | R= 12.8 (A1) sinθ/4.5=sinα/12.8 (M1) θ = 16.3° to vert. (A1) | | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| 6 | (i) | ½.700.20$^2$ or ½.700.15$^2$ | B1 | | either K.E. | |
| | | 700x9.8x400sin5° | B1 | | correct P.E. | |
| | | ½.700.15$^2$+700.9.8.400sin5°=<br>½.700.20$^2$ + W.D. | M1 | | for 4 terms with W.D. | |
| | | W.D. = 178,000 J | A1 | 4 | or 178 kJ | |
| | (ii) | D=200 + 700.9.8sin5° | M1 | | | |
| | | D = 798 N | A1 | | may be implied | |
| | | P = Dx15 = 12,000 = 12kW | A1 | 3 | AG **(11,968W)** | |
| | (iii) | D' = 11,968÷20 = 598 | M1 | | | |
| | | D'-700.9.8sin5°-200 = 700a | M1 | | | |
| | | a = 0.285 ms$^{-2}$ (±) | A1 | 3 | allow 0.283 (from 12kW) | **10** |
| | | **Alternative for false assumption** | | | **of constant acceleration** | |
| | (i) | D-700 x 9.8sin5° = 700a and<br>15$^2$ = 20$^2$ + 2a. 400 | M1 | | (D = 445, a = -0.21875) | |
| | | W.D. = 400xD = 178,000 | A1 | | 2 marks (out of 4) maximum | |

| | | | | | | |
|---|---|---|---|---|---|---|
| 7 | (i) | 50x9.8x2 = Rx3.75 +80x9.8x0.25 | M1 | | moments about D. | |
| | | " | A1 | | SR/no g/ R = 21.3 (M1A1A0) | |
| | | R = 209 N | A1 | 3 | | |
| | (ii) | 130$\bar{x}$ = 50 x 2 + 80 x 4.25 | M1<br>A1 | | moments about BC or FE.....<br>130$\bar{x}$ =80x0.25 + 50x2.5 | |
| | | $\bar{x}$ = 3.385 | A1 | | $\bar{x}$ = 1.115 | |
| | | 130$\bar{y}$ = 50 x 0.125 + 80 x 0.25 | M1<br>A1 | | moments about EC | |
| | | $\bar{y}$ = 0.202 | A1 | | | |
| | | tanθ = 0.615/0.202 | M1 | | | |
| | | θ = 71.8° to the horizontal | A1 | 8 | 71.6° to 72.0° | **11** |

| | | | | | | |
|---|---|---|---|---|---|---|
| 8 | (i) | x = 49cosθ . t | B1 | | | |
| | | y=49sinθ.t - ½.9.8.t$^2$ | B1 | | | |
| | | y=xtanθ - 4.9x$^2$/49$^2$.cos$^2$θ | M1 | | aef (eliminating t) | |
| | | y=xtanθ – x$^2$(1+tan$^2$θ)/490 | A1 | 4 | AG | |
| | (ii) | 30 = 70tanθ – 10(1+tan$^2$θ) | M1 | | | |
| | | tanθ=(70±√3300)÷20 | M1 | | (6.37/0.628) | |
| | | 81.1° | A1 | | $\theta_1$ or $\theta_2$ | |
| | | 32.1° | A1 | 4 | " | |
| | (iii) | x$^2$(1+tan$^2$ θ)/490 = xtanθ | M1 | | set y = 0 | |
| | | x = 490tanθ/(1+tan$^2$ θ) | A1 | | | |
| | | x = 75.0 | A1 | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | x = 221 (220.6) | A1 | | | |
| | | d = 146 m | A1 √ | 5 | √ | **13** |
| | | | | | | |
| | (iii) | Alternatively (1st 2 marks) | | | | |
| | | t=49sinθ/4.9 and (9.88/5.31)<br>x=49cosθ.t | M<br>1 | | <u>$s=ut+½at^2$ and</u><br><u>x=49cosθ.t</u><br>or $R = u^2\sin2\theta/g$ (precise) | |
| | | x = 490sinθcosθ | A1 | | 245sin2θ | |